98. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

A spool, with light thread wound around, is placed upon a rough table so that the thread will emerge from beneath the spool. The thread is passed over a smooth pulley at end of table and a weight attached, the pulley being so adjusted that thread is parallel to surface of table. If friction between spool and table is sufficient to prevent slipping, determine motion of spool and weight. [From problems in Mechanica at Harvard University.]

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

Let x and y be the horizontal and vertical parts of the string at any time t from the beginning of the motion, m the mass of the spool only, m' the mass attached to the thread, ϕ the angle of rotation of the spool in the time t, a and k the radius and radius of gyration of the spool, respectively, g the acceleration of gravity, T the tension in the thread, and F the friction.

The equation of motion of m' is $m'\dfrac{d^2y}{dt^2}=m'g-T$. . .(1).

For the linear and rotary motions of the spool,

$$m\frac{d^2x}{dt^2}=T-F. \;.\;.(2),\quad mk^2\frac{d^2\phi}{dt^2}=-Ta+Fa. \;.\;.(3).$$

Now $dx=ad\phi$. . .(4), and b being the initial length of free string, $x+y=b+a\phi$. . .(5).

From (4), $\dfrac{d^2\phi}{dt^2}=\dfrac{1}{a}\dfrac{d^2x}{dt^2}$. . .(6), and (3) is $mk^2\dfrac{d^2x}{dt^2}=-Ta^2+Fa^2$. . .(7).

From (2), $ma^2\dfrac{d^2x}{dt^2}=Ta^2-Fa^2$. . .(8).

(7) and (8) give $m(a^2+k^2)\dfrac{d^2x}{dt^2}=0$. . .(9), and $T=F$. . .(10).

From (5), $\dfrac{d^2x}{dt^2}+\dfrac{d^2y}{dt^2}=a\dfrac{d^2\phi}{dt^2}$. . .(11), which with (6) gives $\dfrac{d^2y}{dt}=0$. .(12).

(9) gives $\dfrac{d^2x}{dt^2}=0$. . .(13), and (6) then gives $\dfrac{d^2\phi}{dt^2}=0$. . .(14).

Hence, since the system starts from rest, the last three equations show that there is no motion afterwards.